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Structure of A=3 Nuclear Systems Using Realistic Hamiltonians

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Abstract. The structure of A=3 low-energy scattering states is described using the hyperspherical harmonics method with realistic Hamiltonian models, consisting of two- and three-nucleon interactions. Both coordinate and momentum space two-nucleon potential models are considered.

1 Introduction

One of the main ingredients necessary to study few-body nuclear systems is a realistic description of the nuclear interaction. A number of nucleon-nucleon (NN) potentials has been determined in the recent years. They all reproduce the deuteron binding energy and fit a large set of NN scattering data below the pion-production threshold with a χ^2/datum of about 1. Among these potentials, we will consider in the present study only the "phenomenological" model of Ref. [1] (AV18), and a model based on chiral symmetry derived in Ref. [2] (N3LO-Idaho). Among the many features of these two models, we note only that the AV18 is a local NN potential model, with a strong short-range repulsion and tensor component, while the N3LO-Idaho is a non-local NN potential model, with a softer short-range repulsion and tensor component than the AV18. As a consequence of these differences, it is interesting to test these potential models studying light nuclear systems. In these systems, a further contribution to the realistic nuclear Hamiltonian model comes from the three-nucleon interaction (TNI). Several models of TNI's have been proposed. They are mainly based on the exchange of pions among the three nucleons, as the Urbana IX (UIX) TNI [3], which will be considered in the present study. The more recent TNI models studied within the chiral approach [4] and the extension of the UIX model known as the Illinois TNI [5] will be considered in a near future.

A second crucial ingredient in the study of light nuclear systems is the technique used to solve the A-body Schrödinger equation. Several methods have been

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developed in the past years (see Refs. [6, 7] for a review). Among them, we consider in the present study the technique known as the hyperspherical harmonics (HH) method, which will be briefly described in the following section. In Sec. 3, the results for the $n-d$ and $p-d$ scattering lengths will be presented and compared with the available experimental data.

2 The Hyperspherical Harmonics Method

The nuclear wave function for an A-body system can be written as

$$|\Psi\rangle = \Sigma_\mu c_\mu |\Phi_\mu\rangle \ , \tag{1}$$

where $|\Phi_\mu\rangle$ is a suitable complete set of states, and μ is an index denoting the set of quantum numbers necessary to completely determine the basis elements. In the present work, the functions $|\Phi_\mu\rangle$ have been written in terms of HH functions both in configuration-space or in momentum-space [8]. The unknown coefficients c_μ of Eq. (1) are obtained applying the Rayleigh-Ritz (Kohn) variational principle for the bound (scattering) state problem. Then, the matrix elements of the different operators of the Hamiltonian are calculated, working in coordinate- or in momentum-space depending on what is more convenient. Thus, the problem is reduced to an eigenvalue-eigenvector problem (system of algebraic linear equations), which can be solved with standard numerical techniques [9].

3 Results

The $n-d$ and $p-d$ doublet and quartet scattering lengths obtained with the non-local N3LO-Idaho [2] NN interaction, with or without the inclusion of the UIX TNI [3], are given in Table 1, and compared with the available experimental data [10]. Also shown are the results obtained with the local AV18 [1] NN interaction and the AV18/UIX potential model for a comparison [11]. Note that in the case of the N3LO-Idaho/UIX model, the parameter in front of the spin-isospin independent part of the UIX TNI has been rescaled by a factor of 0.384 to fit the triton binding energy. In this way, the triton, ^{3}He, and ^{4}He binding energies are 8.481 MeV, 7.730 MeV, and 28.534 MeV, respectively. Furthermore, the N3LO-Idaho and N3LO-Idaho/UIX results shown in the table are accurate at the 10^{-3} fm level. In fact, the convergence of the HH expansion has been tested with a procedure similar to the one used in Ref. [8] for the A=3 and 4 bound states observables.

From inspection of the table we can conclude that: (i) both the $n-d$ and $p-d$ quartet scattering lengths are very little model-dependent. Also, they are not affected by the inclusion of the TNI. The trend shown by the AV18 and AV18/UIX results has been found also in the case of the non-local N3LO-Idaho and N3LO-Idaho/UIX potential models. (ii) The $n-d$ doublet scattering length is very sensitive to the choice of the NN potential model, when no TNI is included. However, once the TNI is included, and therefore the triton binding energy is well reproduced, $^2a_{nd}$ becomes model-independent. This is a well-known feature, related to the fact that $^2a_{nd}$ and the triton binding energy are linearly correlated

(the so-called Phillips line [12]). (iii) The $p-d$ doublet scattering length is positive and quite model-dependent, if only the two-nucleon interaction is included. Once the TNI is added, ${}^2a_{pd}$ becomes very little and negative. Some model-dependence remains, but the problem of extrapolating to zero energy the experimental results makes impossible any meaningful comparison between theory and experiment.

In conclusion, the application of the HH method to treat the low-energy scattering problem using non-local NN interactions has been found successful. Both $n-d$ and $p-d$ systems have been considered, with the full inclusion of the Coulomb interaction, in the second case. A similar investigation for the A=4 scattering lengths has been reported in Ref. [13]. Further work at higher energies is currently underway.

Table 1. Doublet and quartet $n-d$ and $p-d$ scattering lengths, obtained with different potential models. The experimental $n-d$ scattering lengths are taken from Ref. [10], while the AV18 and AV18/UIX results are taken from Ref. [11].

	AV18	AV18/UIX	N3LO-Idaho	N3LO-Idaho/UIX	Exp.
${}^2a_{nd}$ (fm)	1.27	0.63	1.100	0.623	0.65(4)
${}^4a_{nd}$ (fm)	6.33	6.33	6.342	6.343	6.35(2)
${}^2a_{pd}$ (fm)	1.17	-0.02	0.862	-0.007	
${}^4a_{pd}$ (fm)	13.6	13.7	13.646	13.647	

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